IEA TIMSS 2019

CHAPTER 1

TIMSS 2019 Mathematics Framework

IEA
TIMSS & PIRLS International Study Center
Lynch School of Education
BOSTON COLLEGE

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Mary Lindquist
Ray Philpot
Ina V.S. Mullis
Kerry E. Cotter

Overview

All children can benefit from developing strong skills in and a deep understanding of mathematics. Primarily, learning mathematics improves problem solving skills, and working through problems can teach persistence and perseverance. Mathematics is essential in daily life for such activities as counting, cooking, managing money, and building things. Beyond that, many career fields require a strong mathematical foundation, such as engineering, architecture, accounting, banking, business, medicine, ecology, and aerospace. Mathematics is vital to economics and finance, as well as to the computing technology and software development underlying our technologically advanced and information based world.

This chapter presents the assessment frameworks for the two TIMSS 2019 mathematics assessments:

- TIMSS Mathematics—Fourth Grade
- TIMSS Mathematics—Eighth Grade

As described in the Introduction, the TIMSS 2019 Mathematics Frameworks for the fourth and eighth grades build on TIMSS' 24-year history of assessments every four years since 1995, with this being the seventh assessment in the series.

In general, the fourth and eighth grade frameworks are similar to those used in TIMSS 2015. However, there have been minor updates to particular topics to better reflect the curricula, standards, and frameworks of the participating countries as reported in the *TIMSS 2015 Encyclopedia* (Mullis, Martin, Goh, & Cotter, 2016). Also, because TIMSS 2019 focuses on the transition to eTIMSS, the mathematics frameworks have been updated and are appropriate for both digital and paper assessment formats. The goal is to capitalize on the benefits of computer-based assessment to begin incorporating new and better assessment methods, especially in the applying and reasoning domains (see Chapter 4).

Each of the two assessment frameworks for TIMSS 2019 is organized around two dimensions:

- Content dimension, specifying the subject matter to be assessed
- Cognitive dimension, specifying the thinking processes to be assessed

Exhibit 1.1 shows the target percentage of testing time devoted to each content and cognitive domain for the TIMSS 2019 fourth and eighth grade assessments.

Exhibit 1.1: Target Percentages of the TIMSS 2019 Mathematics Assessment Devoted to Content and Cognitive Domains at the Fourth and Eighth Grades

Fourth Grade

Content Domains	Percentages
Number	50%
Measurement and Geometry	30%
Data	20%

Eighth Grade

Content Domains	Percentages
Number	30%
Algebra	30%
Geometry	20%
Data and Probability	20%

Cognitive Domains	Percentages	
	Fourth Grade	Eighth Grade
Knowing	40%	35%
Applying	40%	40%
Reasoning	20%	25%

The content domains differ for the fourth and eighth grades, reflecting the mathematics widely taught at each grade. There is more emphasis on number at the fourth grade than at the eighth grade. At the eighth grade, two of the four content domains are algebra and geometry. Because these generally are not taught as separable areas in primary school, the introductory or prealgebra topics assessed at the fourth grade are included as part of number. The fourth grade data domain focuses on collecting, reading, and representing data, whereas at the eighth grade it includes more emphasis on interpretation of data, basic statistics, and the fundamentals of probability.

It is important to highlight that TIMSS assesses a range of problem solving situations within mathematics, with about two-thirds of the items requiring students to use applying and reasoning skills. The cognitive domains are the same for both grades, but with a shift in emphasis. Compared to

the fourth grade, the eighth grade has less emphasis on the knowing domain and greater emphasis on the reasoning domain.

Following this brief introduction, the chapter begins with the fourth grade content domains, identifying the three main content domains and the assessment topics within each domain. Next, Chapter 1 continues with the description of the TIMSS Mathematics—Eighth Grade content domains and, then, the descriptions of the cognitive domains for both the fourth and eighth grades.

Mathematics Content Domains—Fourth Grade

Exhibit 1.2 shows the TIMSS Mathematics—Fourth Grade content domains and the target percentages of assessment score points devoted to each. Each content domain consists of topic areas, and each topic area in turn includes several topics. Across the fourth grade mathematics assessment, each topic receives approximately equal weight.

Exhibit 1.2: Target Percentages of the TIMSS 2019 Mathematics Assessment Devoted to Content Domains at the Fourth Grade

Fourth Grade Content Domains	Percentages
Number	50%
Measurement and Geometry	30%
Data	20%

Number

Number provides the foundation of mathematics in primary school. The number content domain consists of three topic areas. The fifty percent of the assessment devoted to number is apportioned as follows:

- Whole numbers (25%)
- Expressions, simple equations, and relationships (15%)
- Fractions and decimals (10%)

Whole numbers are the predominant component of the number domain and students should be able to compute with whole numbers of reasonable size as well as use computation to solve problems. Prealgebra concepts also are part of the TIMSS assessment at the fourth grade, including understanding the concept of variable (unknowns) in simple equations, and initial understandings of relationships between quantities. However, because objects and quantities often do not come in whole numbers, it is also important for students to understand fractions and decimals. Students should be able to compare, add, and subtract familiar fractions and decimals to solve problems.

Whole Numbers

1. Demonstrate knowledge of place value (2-digit to 6-digit numbers); represent whole numbers with words, diagrams, number lines, or symbols; order numbers.
2. Add and subtract (up to 4-digit numbers), including computation in simple contextual problems.
3. Multiply (up to 3-digit by 1-digit and 2-digit by 2-digit numbers) and divide (up to 3-digit by 1-digit numbers), including computation in simple contextual problems.
4. Solve problems involving odd and even numbers, multiples and factors of numbers, rounding numbers (up to the nearest ten thousand), and making estimates.
5. Combine two or more properties of numbers or operations to solve problems in context.

Expressions, Simple Equations, and Relationships

1. Find the missing number or operation in a number sentence (e.g., $17 + w = 29$).
2. Identify or write expressions or number sentences to represent problem situations that may involve unknowns.
3. Identify and use relationships in a well-defined pattern (e.g., describe the relationship between adjacent terms and generate pairs of whole numbers given a rule).

Fractions and Decimals

1. Recognize fractions as parts of wholes or collections; represent fractions using words, numbers, or models; compare and order simple fractions; add and subtract simple fractions, including those set in problem situations. (Fractions may have denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100.)
2. Demonstrate knowledge of decimal place value including representing decimals using words, numbers, or models; compare, order, and round decimals; add and subtract decimals, including those set in problem situations. (Decimals may have one or two decimal places, allowing for computations with money.)

Measurement and Geometry

We are surrounded by objects of different shapes and sizes, and geometry helps us visualize and understand the relationships between shapes and sizes. Measurement is the process of quantifying attributes of objects and phenomena (e.g., length and time).

The two topic areas in measurement and geometry are as follows:

- Measurement (15%)
- Geometry (15%)

At the fourth grade, students should be able to use a ruler to measure length; solve problems involving length, mass, capacity, and time; calculate areas and perimeters of simple polygons; and use cubes to determine volumes. Students should be able to identify the properties and characteristics of lines, angles, and a variety of two- and three-dimensional shapes. Spatial sense is integral to the study of geometry, and students will be asked to describe and draw a variety of geometric figures. They also should be able to analyze geometric relationships and use these relationships to solve problems.

Measurement

1. Measure and estimate lengths (millimeters, centimeters, meters, kilometers); solve problems involving lengths.
2. Solve problems involving mass (gram and kilogram), volume (milliliter and liter), and time (minutes and hours); identify appropriate types and sizes of units and read scales.
3. Solve problems involving perimeters of polygons, areas of rectangles, areas of shapes covered with squares or partial squares, and volumes filled with cubes.

Geometry

1. Identify and draw parallel and perpendicular lines; identify and draw right angles and angles smaller or larger than a right angle; compare angles by size.
2. Use elementary properties, including line and rotational symmetry, to describe, compare, and create common two-dimensional shapes (circles, triangles, quadrilaterals, and other polygons).
3. Use elementary properties to describe and compare three-dimensional shapes (cubes, rectangular solids, cones, cylinders, and spheres) and relate these with their two-dimensional representations.

Data

The explosion of data in today's information society has resulted in a daily bombardment of visual displays of quantitative information. Often the Internet, newspapers, magazines, textbooks, reference books, and articles have data represented in charts, tables, and graphs. Students need to understand that graphs and charts help organize information or categories and provide a way to compare data.

The data content domain consists of two topic areas:

- Reading, interpreting, and representing data (15%)
- Using data to solve problems (5%)

At the fourth grade, students should be able to read and recognize various forms of data displays. Given a simple question, students should be able to collect, organize, and represent the data in graphs and charts to address the question. Students should be able to use data from one or more sources to solve problems.

Reading, Interpreting, and Representing Data

1. Read and interpret data from tables, pictographs, bar graphs, line graphs, and pie charts.
2. Organize and represent data to help answer questions.

Using Data to Solve Problems

1. Use data to answer questions that go beyond directly reading data displays (e.g., solve problems and perform computations using data, combine data from two or more sources, draw conclusions based on data).

Mathematics Content Domains—Eighth Grade

Exhibit 1.3 shows the TIMSS Mathematics—Eighth Grade content domains and the target percentages of assessment score points devoted to each. Each content domain consists of topic areas, and each topic area in turn includes several topics. Across the eighth grade mathematics assessment, each topic receives approximately equal weight.

Exhibit 1.3: Target Percentages of the TIMSS 2019 Mathematics Assessment Devoted to Content Domains at the Eighth Grade

Eighth Grade Content Domains	Percentages
Number	30%
Algebra	30%
Geometry	20%
Data and Probability	20%

Number

At the eighth grade, the thirty percent of the assessment devoted to number consists of three topic areas:

- Integers (10%)
- Fractions and decimals (10%)
- Ratio, proportion, and percent (10%)

Building on the number content domain at the fourth grade, eighth grade students should have developed proficiency with more advanced whole number concepts and procedures as well as extended their mathematical understanding of rational numbers (integers, fractions, and decimals). Students also should understand and be able to compute with integers. Fractions and decimals are an important part of daily life and being able to compute with them requires an understanding of the quantities the symbols represent. Students should understand that fractions and decimals are single entities like whole numbers. A single rational number can be represented with many different written symbols, and students need

to be able to recognize the distinctions among interpretations of rational numbers, convert between them, and reason with them. Students should be able to solve problems involving ratios, proportions, and percents.

Integers

1. Demonstrate understanding of properties of numbers and operations; find and use multiples and factors, identify prime numbers, evaluate positive integer powers of numbers, evaluate square roots of perfect squares up to 144, and solve problems involving square roots of whole numbers.
2. Compute and solve problems with positive and negative numbers, including through movement on the number line or various models (e.g., losses and gains, thermometers).

Fractions and Decimals

1. Using various models and representations, compare and order fractions and decimals, and identify equivalent fractions and decimals.
2. Compute with fractions and decimals, including those set in problem situations.

Ratio, Proportion, and Percent

1. Identify and find equivalent ratios; model a given situation by using a ratio; divide a quantity according to a given ratio.
2. Solve problems involving proportions or percents, including converting between percents and fractions or decimals.

Algebra

The thirty percent of the assessment devoted to algebra is comprised of two topic areas:

- Expressions, operations, and equations (20%)
- Relationships and functions (10%)

Patterns and relationships are pervasive in the world around us and algebra enables us to express these mathematically. Students should be able to solve real world problems using algebraic models and explain relationships involving algebraic concepts. They need to understand that when there is a formula involving two quantities, if they know one quantity, they can find the other either algebraically or by substitution. This conceptual understanding can extend to linear equations for calculations about things that expand at constant rates (e.g., slope). Functions can be used to describe what will happen to a variable when a related variable changes.

Expressions, Operations, and Equations

1. Find the value of an expression or a formula given values of the variables.
2. Simplify algebraic expressions involving sums, products, and powers; compare expressions to determine if they are equivalent.
3. Write expressions, equations, or inequalities to represent problem situations.
4. Solve linear equations, linear inequalities, and simultaneous linear equations in two variables, including those that model real life situations.

Relationships and Functions

1. Interpret, relate and generate representations of linear functions in tables, graphs, or words; identify properties of linear functions including slope and intercepts.
2. Interpret, relate and generate representations of simple non-linear functions (e.g., quadratic) in tables, graphs, or words; generalize pattern relationships in a sequence using numbers, words, or algebraic expressions.

Geometry

Extending the understanding of shapes and measures assessed at the fourth grade, eighth grade students should be able to analyze the properties of a variety of two- and three-dimensional figures and calculate perimeters, areas, and volumes. They should be able to solve problems and provide explanations based on geometric relationships, such as congruence, similarity, and the Pythagorean theorem.

The geometry content domain at the eighth grade consists of one topic area:

- Geometric shapes and measurements (20%)

Geometric Shapes and Measurements

At eighth grade, geometric shapes include circles; scalene, isosceles, equilateral, and right-angled triangles; trapezoids, parallelograms, rectangles, rhombuses, and other quadrilaterals; as well as other polygons including pentagons, hexagons, octagons, and decagons. They also include three-dimensional shapes—prisms, pyramids, cones, cylinders, and spheres. One- and two-dimensional figures can be presented in the Cartesian plane.

1. Identify and draw types of angles and pairs of lines and use the relationships between angles on lines and in geometric figures to solve problems, including those involving the measures of angles and line segments; solve problems involving points in the Cartesian plane.
2. Identify two-dimensional shapes and use their geometric properties to solve problems, including those involving perimeter, circumference, area, and the Pythagorean Theorem.

3. Recognize and draw images of geometric transformations (translations, reflections, and rotations) in the plane; identify congruent and similar triangles and rectangles and solve related problems.
4. Identify three-dimensional shapes and use their geometric properties to solve problems, including those involving surface area and volume; relate three-dimensional shapes with their two-dimensional representations.

Data and Probability

Increasingly, the more traditional forms of data display (e.g., bar graphs, line graphs, pie graphs, pictographs) are being supplemented by an array of new graphic forms (e.g., infographics). By the eighth grade, students should to be able to read and extract the important meaning from a variety of visual displays. It is also important for eighth grade students to be familiar with the statistics underlying data distributions and how these relate to the shape of data graphs. Students should know how to collect, organize, and represent data. Students also should have an initial grasp of some concepts related to probability.

The data and probability content domain contains two topic areas:

- Data (15%)
- Probability (5%)

Data

1. Read and interpret data from one or more sources to solve problems (e.g., interpolate and extrapolate, make comparisons, draw conclusions).
2. Identify appropriate procedures for collecting data; organize and represent data to help answer questions.
3. Calculate, use, or interpret statistics (i.e., mean, median, mode, range) summarizing data distributions; recognize the effect of spread and outliers.

Probability

1. For simple and compound events: a) determine theoretical probability (based on equally likely outcomes, e.g., rolling a fair die) or b) estimate the empirical probability (based on experimental outcomes).

Calculator Use at the Eighth Grade

Continuing the practice of previous TIMSS assessments, at the fourth grade students will not be permitted to use calculators. This includes both paperTIMSS and eTIMSS. At the eighth grade, students will be permitted to use calculators, although the mathematics items are developed to be calculator neutral–do not advantage or disadvantage students whether or not they have calculators. In paperTIMSS, consistent with past TIMSS assessments, students at the eighth grade may bring their own calculators to the assessment. In eTIMSS, students at the eighth grade will have access to a calculator provided as part of the on-screen interface and will not be permitted to bring their own calculators. The on-screen calculator includes the four basic functions (+ , − , ×, ÷) and a square root key. The eventual transition to eTIMSS will result in calculators being standardized.

Mathematics Cognitive Domains—Fourth and Eighth Grades

In order to respond correctly to TIMSS test items, students need to be familiar with the mathematics content being assessed, but they also need to draw on a range of cognitive skills. Describing these skills plays a crucial role in the development of an assessment like TIMSS 2019, because they are vital in ensuring that the survey covers the appropriate range of cognitive skills across the content domains already outlined.

The first domain, *knowing*, covers the facts, concepts, and procedures students need to know, while the second, *applying*, focuses on the ability of students to apply knowledge and conceptual understanding to solve problems or answer questions. The third domain, *reasoning*, goes beyond the solution of routine problems to encompass unfamiliar situations, complex contexts, and multistep problems.

Knowing, applying, and reasoning are exercised in varying degrees when students display their mathematical competency, which goes beyond content knowledge. These TIMSS cognitive domains encompass the competencies of problem solving, providing a mathematical argument to support a strategy or solution, representing a situation mathematically (e.g., using symbols and graphs), creating mathematical models of a problem situation, and using tools such as a ruler or a calculator to help solve problems.

The three cognitive domains are used for both grades, but the balance of testing time differs, reflecting the difference in age and experience of students in the two grades. For the fourth and eighth grades, each content domain will include items developed to address each of the three cognitive domains. For example, the number domain will include knowing, applying, and reasoning items as will the other content domains.

Exhibit 1.4 shows the target percentages of testing time devoted to each cognitive domain for the fourth and eighth grade assessments.

Exhibit 1.4: Target Percentages of the TIMSS 2019 Mathematics Assessment Devoted to Cognitive Domains at the Fourth and Eighth Grades

Cognitive Domains	Percentages	
	Fourth Grade	Eighth Grade
Knowing	40%	35%
Applying	40%	40%
Reasoning	20%	25%

Knowing

Facility in applying mathematics, or reasoning about mathematical situations, depends on familiarity with mathematical concepts and fluency in mathematical skills. The more relevant knowledge a student is able to recall and the wider the range of concepts he or she understands, the greater the potential for engaging in a wide range of problem solving situations.

Without access to a knowledge base that enables easy recall of the language and basic facts and conventions of number, symbolic representation, and spatial relations, students would find purposeful mathematical thinking impossible. Facts encompass the knowledge that provides the basic language of mathematics, as well as the essential mathematical concepts and properties that form the foundation for mathematical thought.

Procedures form a bridge between more basic knowledge and the use of mathematics for solving problems, especially those encountered by many people in their daily lives. In essence, a fluent use of procedures entails recall of sets of actions and how to carry them out. Students need to be efficient and accurate in using a variety of computational procedures and tools. They need to see that particular procedures can be used to solve entire classes of problems, not just individual problems.

Recall	Recall definitions, terminology, number properties, units of measurement, geometric properties, and notation (e.g., $a \times b = ab$, $a + a + a = 3a$).
Recognize	Recognize numbers, expressions, quantities, and shapes. Recognize entities that are mathematically equivalent (e.g., equivalent familiar fractions, decimals, and percents; different orientations of simple geometric figures).
Classify/Order	Classify numbers, expressions, quantities, and shapes by common properties.
Compute	Carry out algorithmic procedures for +, −, ×, ÷, or a combination of these with whole numbers, fractions, decimals, and integers. Carry out straightforward algebraic procedures.
Retrieve	Retrieve information from graphs, tables, texts, or other sources.
Measure	Use measuring instruments; and choose appropriate units of measurement.

Applying

The applying domain involves the application of mathematics in a range of contexts. In this domain, the facts, concepts, and procedures as well as the problems should be familiar to the student. In some items aligned with this domain, students need to apply mathematical knowledge of facts, skills, and procedures or understanding of mathematical concepts to create representations. Representation of ideas forms the core of mathematical thinking and communication, and the ability to create equivalent representations is fundamental to success in the subject.

Problem solving is central to the applying domain, with an emphasis on more familiar and routine tasks. Problems may be set in real life situations, or may be concerned with purely mathematical questions involving, for example, numeric or algebraic expressions, functions, equations, geometric figures, or statistical data sets.

Determine	Determine efficient/appropriate operations, strategies, and tools for solving problems for which there are commonly used methods of solution.
Represent/Model	Display data in tables or graphs; create equations, inequalities, geometric figures, or diagrams that model problem situations; and generate equivalent representations for a given mathematical entity or relationship.
Implement	Implement strategies and operations to solve problems involving familiar mathematical concepts and procedures.

Reasoning

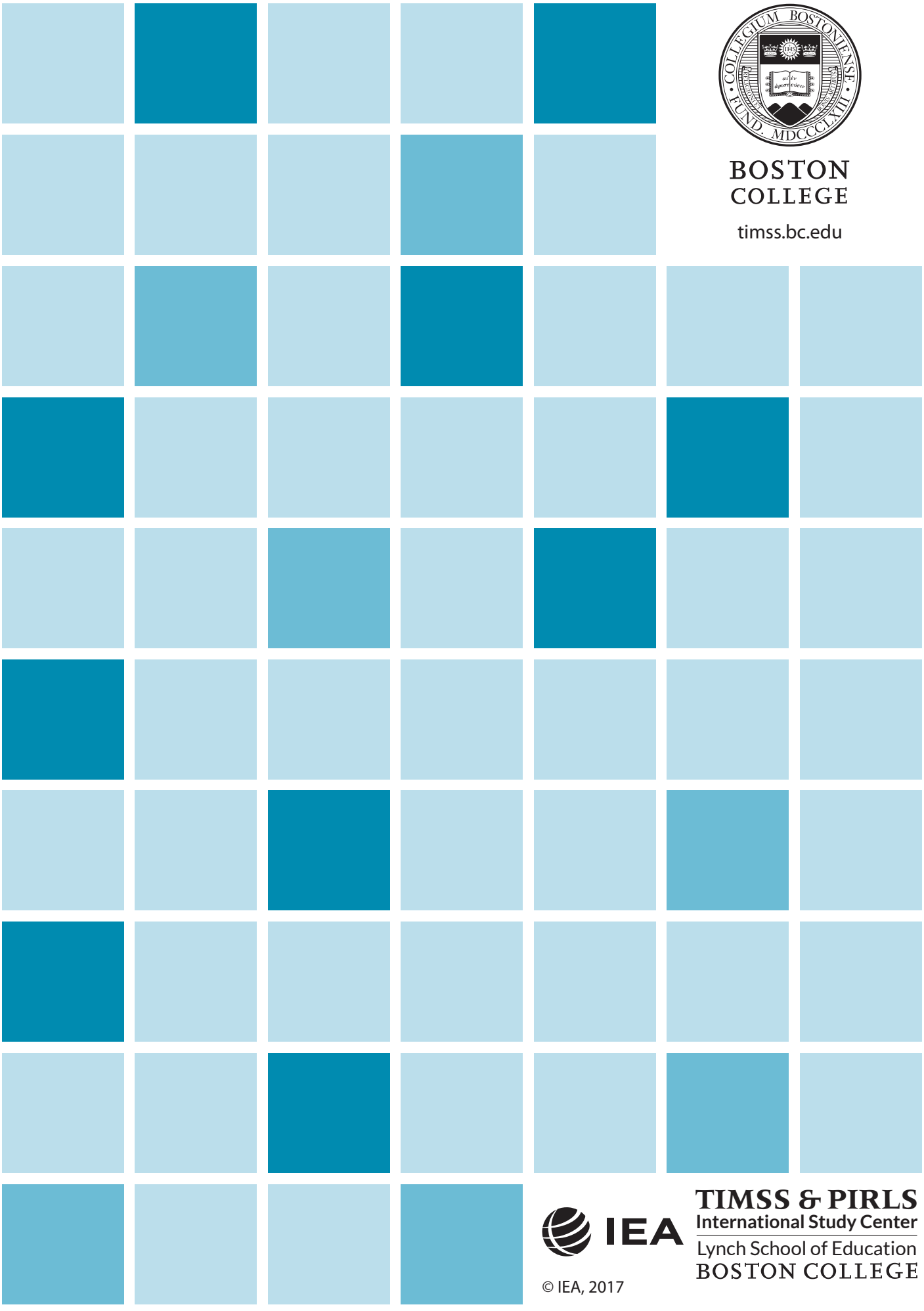
Reasoning mathematically involves logical, systematic thinking. It includes intuitive and inductive reasoning based on patterns and regularities that can be used to arrive at solutions to problems set in novel or unfamiliar situations. Such problems may be purely mathematical or may have real life settings. Both types of items involve transferring knowledge and skills to new situations; and interactions among reasoning skills usually are a feature of such items.

Even though many of the cognitive skills listed in the reasoning domain may be drawn on when thinking about and solving novel or complex problems, each by itself represents a valuable outcome of mathematics education, with the potential to influence learners' thinking more generally. For example, reasoning involves the ability to observe and make conjectures. It also involves making logical deductions based on specific assumptions and rules, and justifying results.

Analyze	Determine, describe, or use relationships among numbers, expressions, quantities, and shapes.
Integrate/Synthesize	Link different elements of knowledge, related representations, and procedures to solve problems.
Evaluate	Evaluate alternative problem solving strategies and solutions.
Draw Conclusions	Make valid inferences on the basis of information and evidence.
Generalize	Make statements that represent relationships in more general and more widely applicable terms.
Justify	Provide mathematical arguments to support a strategy or solution.

References

Mullis, I.V.S., Martin, M.O., Goh, S., & Cotter, K. (Eds.). (2016). *TIMSS 2015 encyclopedia: Education policy and curriculum in mathematics and science*. Retrieved from Boston College, TIMSS & PIRLS International Study Center website: http://timssandpirls.bc.edu/timss2015/encyclopedia/



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